1) Να αναλυθεί η καμπύλη

$$x^2+6xy+y^2+6x+2y-1=0 \quad ①$$

ΛΥΣΗ

$A=1,\ B=3,\ \Gamma=1,\ \Delta=3,\ E=1,\ Z=-1$

$$\left.\begin{aligned} &\Sigma = A+\Gamma = 2>0 \\ &d=\begin{vmatrix} A & B \\ B & \Gamma \end{vmatrix} = -8<0 \\ &D=\begin{vmatrix} A & B & \Delta \\ B & \Gamma & E \\ \Delta & E & Z \end{vmatrix} = 16 \neq 0 \end{aligned}\right\} \text{Λόγω } d<0,\ D\neq 0 \text{ Υπερβολή}$$

Στρέφαμε κατά φ το σύστημα

$$\varepsilon\varphi 2\phi = \frac{2B}{A-\Gamma} = \frac{6}{1-1} = \infty \Rightarrow 2\varphi = \frac{\pi}{2} \Rightarrow \varphi = \frac{\pi}{4} \text{ rad}$$

Άρα, $x = x'\sigma\upsilon\nu\phi - y'\cdot\eta\mu\phi = \frac{\sqrt{2}}{2}(x'-y')$ ②

$y = x'\cdot\eta\mu\phi + y'\cdot\sigma\upsilon\nu\phi = \frac{\sqrt{2}}{2}(x'+y')$ ③

Άρα ① είναι:

$$\left[\frac{\sqrt{2}}{2}(x'-y')\right]^2 + 6\left(\frac{\sqrt{2}}{2}(x'-y')\right)\left(\frac{\sqrt{2}}{2}(x'+y')\right) + \left[\frac{\sqrt{2}}{2}(x'+y')\right]^2 +$$

$$+6\left(\frac{\sqrt{2}}{2}(x'-y')\right) + 2\left(\frac{\sqrt{2}}{2}(x'+y')\right) - 1 = 0 \Rightarrow$$

$$\Rightarrow \frac{2}{4}(x'-y')^2 + 6\frac{2}{4}(x'^2-y'^2) + \frac{2}{4}(x'+y')^2 + 6\frac{\sqrt{2}}{2}(x'-y') + 2\frac{\sqrt{2}}{2}(x'+y') - 1 = 0$$

$$\Rightarrow \left(\frac{2}{4}+\frac{12}{4}+\frac{2}{4}\right)x'^2 + \cancel{\left(\frac{4}{4}-\frac{4}{4}\right)}^{\to 0}x'y' + \left(\frac{2}{4}-\frac{12}{4}+\frac{2}{4}\right)y'^2 + \frac{8\sqrt{2}}{2}x - \frac{4\sqrt{2}}{2}y - 1 = 0$$

$$\Rightarrow 4x'^2 - 2y'^2 + 4\sqrt{2}x - 2\sqrt{2}y - 1 = 0 \Rightarrow$$

$$\Rightarrow 4\left(x'^2 + 2\frac{\sqrt{2}}{2}x' + \frac{2}{4}\right) - 2\left(y'^2 + 2\frac{\sqrt{2}}{2}y' + \frac{2}{4}\right) = 1+2-1$$

$$\frac{(x'+\sqrt{2}/2)^2}{2/4} - \frac{(y'+\sqrt{2}/2)^2}{2/2} = 1 \Rightarrow \frac{(x'+\sqrt{2}/2)^2}{1/2} - (y'+\sqrt{2}/2)^2 = 1$$

Θέτουμε

$$\left.\begin{aligned} X &= x' + \frac{\sqrt{2}}{2} \\ Y &= y' + \frac{\sqrt{2}}{2} \end{aligned}\right\} \quad \begin{aligned} x' &= -\frac{\sqrt{2}}{2} + X \\ y' &= -\frac{\sqrt{2}}{2} + Y \end{aligned} \quad \Bigg| \text{ Άρα, } \frac{X^2}{1/2} - \frac{Y^2}{1} = 1$$

Αφού, κάναμε την παράλληλη μεταφορά

Άρα, το new σύστημα είναι

$$Ox'y' \to O'\left(x' = -\frac{\sqrt{2}}{2},\ y' = -\frac{\sqrt{2}}{2}\right)$$

Άρα, στις ② και ③ έχουμε

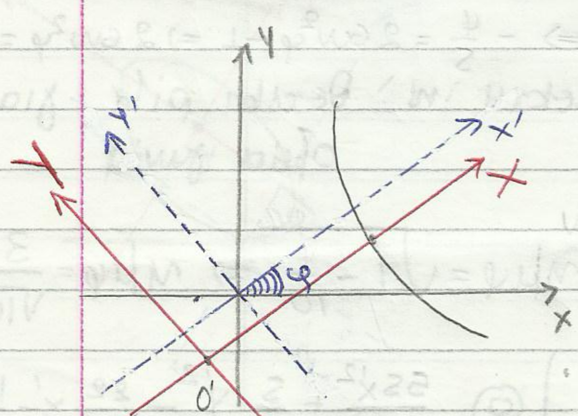
$$\left.\begin{aligned} x &= \frac{\sqrt{2}}{2} \cdot 0 \Rightarrow x = 0 \\ y &= \frac{\sqrt{2}}{2}(-\sqrt{2}) \Rightarrow y = -1 \end{aligned}\right\} O'(0,-1)$$

ή Πιο καλός τρόπος

$$D = \begin{vmatrix} A & B & \Delta \\ B & \Gamma & E \\ \Delta & E & Z \end{vmatrix} = 16 \neq 0, \text{ Άρα } \begin{cases} A\cdot\alpha + B\cdot\beta + \Delta = 0 \\ B\cdot\alpha + \Gamma\cdot\beta + E = 0 \end{cases} \Rightarrow \begin{cases} \alpha + 3\beta + 3 = 0 \\ 3\alpha + \beta + 1 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \alpha = 0 \\ \beta = -1 \end{cases} O'(0,-1)$$

ΣΧΗΜΑ:



Έπειτα, οι ασύμπτωτες βρίσκονται με τον εξής τρόπο.

$$\frac{X^2}{1/2} - \frac{Y^2}{1} = 0 \Rightarrow X = \pm\frac{Y}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}X - Y = 0 \ ④ \quad \text{ή} \quad \sqrt{2}X + Y = 0 \ ⑤$$

Όμως τις ασύμπτωτες τις θέλω στο αρχικό σύστημα

Άρα, ⑤ $\sqrt{2}X + Y = 0 \Rightarrow \sqrt{2}\left(x' + \frac{\sqrt{2}}{2}\right) + y' + \frac{\sqrt{2}}{2} = 0 \Rightarrow$

$$\Rightarrow \sqrt{2}x' + 1 + y' + \frac{\sqrt{2}}{2} = 0 \Rightarrow \sqrt{2}\left(\frac{x+y}{\sqrt{2}}\right) + 1 + \frac{x-y}{\sqrt{2}} + \frac{\sqrt{2}}{2} = 0 \Rightarrow$$

$$\Rightarrow (\varepsilon_1)\ \frac{\sqrt{2}-1}{\sqrt{2}}x + \frac{\sqrt{2}+1}{\sqrt{2}}y + \frac{\sqrt{2}+1}{\sqrt{2}} = 0$$

Ομοίως, και για την (ε_2)